Corso di Laurea a ciclo Unico in Ingegneria Edile - Architettura

Geotecnica e Laboratorio

Resistenza al taglio delle terre – parte 1

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The Mohr circle and Failure Theories

If the load or stress in a foundation or earth slope is increased until the deformations become unacceptably large, we say that soil has FAILED.

In this case we are referring to the **STRENGTH** of the soil, which is really the **maximum or ultimate stress** the material can sustain.

In geotechnical engineering we are generally concerned with the shear strength of soils because in most of our problems in foundations and earthwork engineering, failure results from excessive applied shear stresses.
STRESS AT A POINT

The concept of stress at a point in a soil is really fictitious.

The point of application of a force, within a soil mass, could be on a particle or in a void.

Clearly, a void cannot support any force, but if the force were applied to a particle, the stress could be extremely large.

When we speak about stress in the context of soil materials we are really speaking about a force per unit area, in which the considered area is the gross cross-sectional or engineering area.

This area contains both grain-to-grain contacts, as well as voids.
Stress at a point

Consider a soil mass that is acted upon by a set of generic forces $F_1$, $F_2$, ..., $F_n$. 

![Diagram showing forces acting on a soil mass](image)
Let’s assume that these forces only act in a two-dimensional plane.

We could resolve these forces into components on a small element at any point within the soil mass, such as point O in Figure 1.

The resolution of these forces into normal and shear components acting, for example, on a plane passing through point O at an angle $\alpha$ from the horizontal, is shown in Figure 2a, which is an expanded view of a small element at point O.
For convenience our sign convention has \textbf{compressive forces and stresses positive} because most normal stresses in geotechnical engineering are compressive.

This convention then requires that a positive shear stress produce counter clockwise couples on our element.

Put another way: positive shears produce clockwise moments about a point just outside the element.

Clockwise angles are also taken to be positive. These conventions are the opposite of those normally assumed in structural mechanics.
Let’s assume that the distance AC along the inclined plane has unit length and the figure has a unit depth perpendicular to the plane of the Figure.

Thus the vertical plane BC has the dimension of \((1 \sin \alpha)\) while the horizontal dimension AB has a dimension equal to \((1 \cos \alpha)\).

At equilibrium, the sum of the forces in any direction must be zero.
Summing in horizontal and vertical directions, we obtain:

\[ \sum (F_h) = H - T \cdot \cos \alpha - N \cdot \sin \alpha = 0 \]  \hspace{1cm} (1a)

\[ \sum (F_v) = V + T \cdot \sin \alpha - N \cdot \cos \alpha = 0 \]  \hspace{1cm} (1b)

**Figure 2a**

Stress at a point
Dividing the forces in eq.1 by the areas upon which they act, we obtain the normal and shear stresses.

We shall denote the horizontal normal stress by $\sigma_x$ and the vertical normal stress by $\sigma_y$; the stresses on the $\alpha$-plane are the normal stress $\sigma_\alpha$ and the shear stress $\tau_\alpha$:

\begin{align}
\sigma_x \cdot \sin \alpha - \tau_\alpha \cdot \cos \alpha - \sigma_\alpha \cdot \sin \alpha &= 0 \\
\sigma_y \cdot \cos \alpha + \tau_\alpha \cdot \sin \alpha - \sigma_\alpha \cdot \cos \alpha &= 0
\end{align}  \tag{2a} (2b)
Solving eq. 2a and 2b simultaneously for $\sigma_a$ and $\tau_a$ we obtain:

\[
\sigma_\alpha = \sigma_x \cdot \sin^2 \alpha + \sigma_y \cdot \cos^2 \alpha = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cdot \cos 2\alpha
\]  

(3)

\[
\tau_\alpha = (\sigma_x - \sigma_y) \cdot \sin \alpha \cdot \cos \alpha = \frac{\sigma_x - \sigma_y}{2} \cdot \sin 2\alpha
\]  

(4)

If you square and add these equations, you will obtain the equation for a circle with a radius of $(\sigma_x - \sigma_y)/2$ and centre at $[(\sigma_x + \sigma_y)/2, 0]$.

When this circle is plotted in $\tau - \sigma$ space (Fig. 3b) for the element in Fig. 3a, it is known as the **Mohr circle of stress**.
Stress at a point

It represents the state of stress at a point at equilibrium, and it applies to any material, not just soil.

Note that the scales for $\tau$ and $\sigma$ have to be the same to obtain a circle from these equations.

Since the vertical and horizontal planes in Fig. 2a and 3a have no shearing stresses acting on them, they are by definition principal planes.
Thus the stresses $\sigma_x$ and $\sigma_y$ are really **principal stresses**.

Principal stresses act on planes where $\tau = 0$.

The stress with the largest magnitude is called the **major principal stress** and denoted by the symbol $\sigma_1$.

The smallest principal stress is called the **minor principal stress**, $\sigma_3$ and the stress in the third dimension is the **intermediate principal stress**, $\sigma_2$. 

**Figure 3c**
Stress at a point

In Fig. 3b, $\sigma_2$ is neglected since our derivation was for two-dimensional (plane stress) conditions.

We could, however, draw two additional Mohr circles for $\sigma_1$ and $\sigma_2$ and $\sigma_2$ and $\sigma_3$ to make a complete Mohr diagram (Fig.3c).

Figure 3 b,c
Now we can write Equations 3 and 4 in terms of principal stresses.

\[
\sigma_\alpha = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cdot \cos 2\alpha 
\]

\[
\tau_\alpha = \frac{\sigma_1 - \sigma_3}{2} \cdot \sin 2\alpha 
\]
Here we have arbitrarily assumed that $\sigma_x = \sigma_1$ and $\sigma_y = \sigma_3$.

You should verify that the coordinates of $(\sigma_a, \tau_a)$ in Fig.3b can be determined by Eq.5 and 6.

From these equations, also verify that the coordinates of the centre of the circle are $[(\sigma_1 + \sigma_3)/2, 0]$, and that the radius is $[(\sigma_1 - \sigma_3)/2]$.

It is now possible to calculate the normal stress $\sigma_a$ and shear stress $\tau_a$ on any plane $\alpha$, as long as we know the principal stresses.

In fact, we could almost as easily derive equations for the general case where $\sigma_x$ and $\sigma_y$ are not principal planes.

These equations are known as the double angle equations.
The analytical procedure is sometimes awkward to use in practice because of the double angles; we prefer to use a graphical procedure based on a unique point on the Mohr circle called the pole or the origin of planes.

This point has a very useful property: any straight line drawn through the pole will intersect the Mohr circle at a point which represents the state of stress on a plane inclined at the same orientation in space as the line.

This concept means that if you know the state of stress, $\sigma$ and $\tau$, on some plane in space, you can draw a line parallel to that plane through the coordinates of $\sigma$ and $\tau$ on the Mohr circle.
The pole then is the point where that line intersects the Mohr circle.

Once the pole is known, the stresses on any plane can readily be found by simply drawing a line from the pole parallel to that plane; the coordinates of the point of intersection with the Mohr circle determine the stresses that plane.
example n.1

stresses on an element Fig. a

Find normal stress $\sigma_a$ and shear stress $t$ on the plane inclined at $\alpha = 35^\circ$ from the horizontal reference plane
example n.2

stresses on the same element rotated 20°

Find normal stress \( \sigma_a \) and shear stress \( \tau \) on the plane inclined at \( \alpha = 35^\circ \) from the horizontal reference plane.

Note that the element is rotated 20° from the horizontal.
Stress-strain relationships and failure criteria

The stress-strain curve for mild steel is shown in Fig. 4a.

The initial portion up to the proportional limit (yield point) is linearly elastic. This means that the material will return to its original shape when the stress is released, as long as the applied stress is below the yield point.
It is possible, however, for a material to have a **nonlinear stress-strain curve and still be elastic**, as shown in Fig.4b.

Note that both these stress-strain relationships are independent of time.

If time is a variable, then the material is called **visco-elastic**.

Some real materials such as most soils and polymers are visco-elastic.
Why don’t we use a visco-elastic theory to describe the behaviour of soils? Soils have a highly nonlinear stress-strain-time behaviour, and unfortunately only a mathematically well-developed linear theory of visco-elasticity is available.

Note that so far we’ve said nothing about failure or yield.

Even linearly elastic materials yield, as indicated in Fig.4a, if sufficient stress is applied.
At the proportional limit the material becomes **plastic or yields plastically**.

Real materials behaviour can be idealized by several plastic stress-strain relationships (Fig.4c,d,f).

Perfectly plastic materials (Fig.4c), sometimes called **rigid-plastic**, can be treated relatively easily mathematically, and thus are popular subjects of study by mechanicians and mathematicians.

A more realistic stress-strain relationship is elasto-plastic (Fig.4d).

**Figure 4**
The material is **linearly elastic** up to the **yield point** $\sigma_y$; then it becomes **perfectly plastic**.

Both perfectly plastic and elasto-plastic materials continue to strain even without any additional stress applied.

The stress-strain curve for mild steel can be approximated by an elasto-plastic stress-strain curve, and this theory is very useful in working, punching, and machining of metals.
Sometimes materials such as cast iron, concrete, and a lot of rocks are **brittle**, in that they exhibit very little strain as the stress increases. Then, at some point, the material suddenly collapses or crushes (Fig. 4e).

More complex but also realistic for many materials are the stress-strain relations shown in Fig. 4f.

Work-hardening materials, as the name implies, become stiffer (higher modulus) as they are strained or “worked.”
The little hump in the stress-strain curve for mild steel after yield (Fig. 4a) is an example of work-hardening.

Many soils are also work-hardening, for example, compacted clays and loose sands.

Work-softening materials (Fig. 4f) show a decrease in stress as they are strained beyond a peak stress.
Sensitive clay soils and dense sands are examples of work-softening materials.

At what point on the stress-strain curve do we have failure?

**We could call the yield point “failure” if we wanted to.**

In some situations, if a material is stressed to its yield point, the strains or deflections are so large that for all practical purposes the material has failed.

This means that the material cannot satisfactorily continue to carry the applied loads. The stress at “failure” is often very arbitrary, especially for nonlinear materials.
With brittle-type materials, however, there is no question when failure occurs—it’s obvious. Even with work-softening materials (Fig. 4f), the peak of the curve or the maximum stress is usually defined as failure.

On the other hand, with some plastic materials it may not be obvious.

Where would you define failure if you had a work-hardening stress-strain curve (Fig. 4f).
With materials such as these, we usually define failure at some arbitrary percent strain, for example, 15% or 20%, or at a strain or deformation at which the function of the structure might be impaired.

Now we can also define the strength of a material. It is the maximum or yield stress or the stress at some strain which we have defined as “failure.”

**Figure 4**
There are many ways of defining failure in real materials; in other words, there are many failure criteria.

Most of the criteria don’t work for soils, and in fact the one we do use doesn’t always work so well either.

The most common failure criterion applied to soils is the Mohr-Coulomb failure criterion.
Coulomb is well known from coulombic friction, electrostatic attraction and repulsion, among other things.

Around the turn of last century, Mohr (1900) hypothesized a criterion of failure for real materials in which he stated that materials fail when the shear stress on the failure plane at failure reaches some unique function of the normal stress on that plane, or:

\[ \tau_{ff} = f(\sigma_{ff}) \]  

where \( \tau \) is the shear stress and \( \sigma \) is the normal stress.

The first subscript \( f \) refers to the plane on which the stress acts (in this case the failure plane) and the second \( f \) means “at failure.”
\( \tau_{ff} \) is called shear strength of the material. The relationship (Eq.7) is shown in Fig.5a.

Fig.5b shows an element at failure with the principal stresses that caused failure and the resulting normal and shear stresses on the failure plane.

We will assume that a failure plane exists, which is not a bad assumption for soils, rocks, and many other materials.
We won’t worry now about how the principal stresses at failure are applied to the element (test specimen or representative element in the field) or how they are measured.

If we know the principal stresses at failure, we can draw a Mohr circle to represent this state of stress for this particular element.
We could carried out several tests to failure or measure failure stresses in several elements at failure, and draw Mohr circles for each element or test at failure. (Figure 6)

Note that only the top half of the Mohr circles are drawn, which is conventionally done in soil mechanics for convenience only.

Since the Mohr circles are determined at failure, it is possible to construct the limiting or failure envelope of the shear stress.

This envelope, called **Mohr failure envelope**, expresses the functional relationship between shear stress $\tau_{ff}$ and normal stress $\sigma_{ff}$ at failure (Eq.7).

$$\tau_{ff} = f \left(\sigma_{ff}\right)$$
Any circle lying **below** the Mohr failure envelope (circle A) represents a **stable condition**. **Failure occurs** only when the combination of shear and normal stress is such that the Mohr circle is **tangent** to the Mohr failure envelope.

**Circles lying above Mohr failure envelope (circle B) cannot exist.** The material would fail before reaching these states of stress.

If this **envelope is unique for a given material**, the point of tangency of Mohr failure envelope gives the stress conditions on failure plane at failure.

Using the pole method, we can determine the angle of the failure plane from the point of tangency of the Mohr circle and the Mohr failure envelope.
Mohr failure hypothesis: the point of tangency defines the angle of the failure plane in the element or test specimen. You should distinguish this hypothesis from the Mohr failure theory.

The Mohr failure hypothesis is illustrated in Fig. 7a for the element at failure shown in Fig. 7b.

Stated another way: the Mohr failure hypothesis states that the point of tangency of the Mohr failure envelope with the Mohr circle at failure determines the inclination of the failure plane.
Even though in soil mechanics we **draw only the top half** of the Mohr circle, there is a bottom half, and also a “bottom-half” Mohr failure envelope.

This also means, if the Mohr failure hypothesis is valid, that it is equally likely that a failure plane will form at an angle of $-\alpha_f$ (Fig. 7a).

It is nonuniform stress conditions on the ends of a test specimen and small inhomogeneities within the specimen itself that we think cause a single failure plane to often form in a test specimen.
Ever wonder why a cone forms at failure in the top and bottom of a **concrete cylinder** when it is failed in compression?

Shear stresses between the testing machine and specimen caps cause nonuniform stresses to develop within the specimen.

If everything is homogeneous and uniform stress conditions are applied to a specimen, then **multiple failure planes** form at conjugate angles, $\pm \alpha_f$ (Fig. 7c).
Coulomb (1776) was concerned with military defence works such as revetments and fortress walls. These constructions were built by rule of thumb and unfortunately for the French military defences many of these works failed. Coulomb became interested in the problem of the lateral pressures exerted against retaining walls, and he devised a system for analysis of earth pressures against retaining structures that is still used today.
One of the things he needed for design was shearing strength of the soil.

Being also interested in the sliding friction characteristics of different materials, he set up a device for determining the shear resistance of soils.

He observed that there was a stress-independent component of shear strength and a stress-dependent component.

The stress-dependent component is similar to sliding friction in solids, so he called this component the angle of shear strength, denoting it by the \( \phi \) symbol. The other component seemed to be related to the intrinsic cohesion of the material and it is commonly denoted by the symbol \( c \).

Coulomb’s equation is, then:

\[
\tau_f = c + \sigma \tan \phi
\]
MOHR-COULOMB FAILURE CRITERION

where \( \tau_f \) is the shear strength of the soil, \( \sigma \) is the applied normal stress, and \( \phi \) and \( c \) are the strength parameters of the soil.

This relationship gives a straight line and is, therefore, easy to work with.

Neither \( \phi \), nor \( c \) are inherent properties of the material; they are dependent on the conditions operative in the test.

We could plot the results of a shear test on soil to obtain the strength parameters \( \phi \) and \( c \) (Fig.8).
MOHR-COULOMB FAILURE CRITERION

Note that either strength parameter could be zero for any particular stress condition; that is, \( \tau = c \) when \( \phi = 0 \), or \( \tau = \sigma \tan \phi \), when \( c = 0 \).

These relationships are valid for certain specific test conditions for some soils.

Although who first did so is unknown, it would seem reasonable to combine the Coulomb equation, Eq.8, with the Mohr failure criterion, Eq.7.
Engineers traditionally prefer to work with straight lines since anything higher than a first-order equation (straight line) is too complicated!!!

So the natural thing to do was to straighten out that curved Mohr failure envelope, or at least approximate the curve by a straight line over some given stress range; then the equation for that line in terms of the Coulomb strength parameters could be written.
Thus the Mohr-Coulomb strength criterion was born; it is the most popular strength criterion applied to soils.

Mohr-Coulomb criterion can be written as:

$$\tau_{\text{ff}} = c + \sigma_{\text{ff}} \tan \phi$$  \hspace{1cm} (9)

This is a simple, easy-to-use criterion that has many distinct advantages over other failure criteria.
It is the only failure criterion which predicts the stresses on the failure plane at failure, and since soil masses have been observed to fail on rather distinct surfaces, we would like to be able to estimate the state of stress on potential sliding surfaces.

So the Mohr-Coulomb criterion is very useful for analyses of the stability of earth slopes and foundation.

Let’s look a little more carefully at some Mohr circles, both before failure and at failure.

First, if we know the angle of inclination of the Mohr failure envelope or have determined it from laboratory tests, then it is possible to write the angle of the failure plane $\alpha_f$ in terms of the slope $\phi$ of the Mohr failure envelope.
To do this, we have to invoke the Mohr failure hypothesis. Then the failure angle measured relative to the plane of the major principal stress is:

\[ \alpha_f = 45^\circ + \frac{\phi}{2} \] (10)
Let’s look at a soil element subjected to principal stresses < than the failure stresses (9a).

\( \tau_f \) is mobilized shear resistance on the potential failure plane, and \( \tau_{ff} \) is shear strength available (shear stress on failure plane at failure).

Not having reached failure yet, there is some reserve strength remaining and this really is a definition of the factor of safety in the material.

Figure 9
Now, if the stresses increase so that failure occurs, then the Mohr circle becomes tangent to the Mohr failure envelope.

According to the Mohr failure hypothesis, failure occurs on the plane inclined at \( \alpha_f \) and with shear stress on that plane of \( \tau_{ff} \).

This is not the largest or maximum shear stress in the element!!!

The maximum shear stress acts on the plane inclined at 45° and is equal to:

\[
\tau_{max} = \frac{\sigma_{1f} - \sigma_{3f}}{2} > \tau_{ff}
\]

(12)
Why doesn’t failure occur on the 45° plane?

It cannot because on that plane the shear strength available is greater than \( \tau_{\text{max}} \) so failure cannot occur.

This condition is represented by the distance from the maximum point on the Mohr circle up to the Mohr failure envelope in Fig. 9b.

That would be the shear strength available when the normal stress on the 45° plane was \( (\sigma_{1f} + \sigma_{3f})/2 \).
The only exception would be when shear strength is independent of normal stress, i.e., when Mohr failure envelope is horizontal and $\phi = 0$.

Fig.9c is valid for UU triaxial tests.

**Such materials are called purely cohesive for obvious reasons.**

Failure theoretically occurs on the $45^\circ$ plane (it doesn’t really).

The shear strength is $\tau_f$, and the normal stress on the theoretical failure plane at failure is $(\sigma_{1f} + \sigma_{3f})/2$. 
Now we write
the Mohr-
Coulomb failure
criterion in
terms of
principal
stresses at
failure, rather
than in terms of
\( \tau_{ff} \) and \( \sigma_{ff} \).

Look at Fig. 10
and note that
\( \sin \phi = \frac{R}{D} \) or:

\[
\sin \phi = \frac{\frac{\sigma_{1f} - \sigma_{3f}}{2}}{\frac{\sigma_{1f} + \sigma_{3f}}{2} + c \cdot \cot \phi}
\]

\[
(\sigma_{1f} - \sigma_{3f}) = (\sigma_{1f} + \sigma_{3f}) \cdot \sin \phi + 2 \cdot c \cdot \cos \phi
\]
If $c = 0$ then \[ (\sigma_{1f} - \sigma_{3f}) = (\sigma_{1f} + \sigma_{3f}) \sin \phi \] which can be written:

$$\sin \phi = \frac{\sigma_{1f} - \sigma_{3f}}{\sigma_{1f} + \sigma_{3f}}$$

$$\frac{\sigma_1}{\sigma_3} = \frac{1 + \sin \phi}{1 - \sin \phi}$$

$$\frac{\sigma_3}{\sigma_1} = \frac{1 - \sin \phi}{1 + \sin \phi}$$

**Figure 10**
MOHR-COULOMB FAILURE CRITERION

\[
\frac{\sigma_1}{\sigma_3} = \tan^2\left(45^\circ + \frac{\phi}{2}\right)
\]

\[
\frac{\sigma_3}{\sigma_1} = \tan^2\left(45^\circ - \frac{\phi}{2}\right)
\]

**Figure 10**
Eq. 14 through 17 are called the obliquity relationships because the maximum inclination, or obliquity, of Mohr failure envelope occurs where $c = 0$.

These four equations are only valid where $c = 0$.

Inspection of these equations and Fig. 10 shows that the coordinates of the point of tangency of the Mohr failure envelope and the Mohr circle ($\sigma_{ff}$, $\tau_{ff}$) are the stresses on the plane of maximum obliquity in the soil element.

In other words, the ratio $\tau_{ff}/\sigma_{ff}$ is a maximum on this plane.
This plane is not the plane of maximum shear stress.

On that plane ($\alpha = 45^\circ$), the obliquity will be $< \text{the maximum value since } \tau_{\text{max}}/[(\sigma_1 + \sigma_3)/2] < (\tau_{\text{ff}}/\sigma_{\text{ff}})$

The obliquity relationships are very useful for evaluating triaxial test data and in theories of lateral earth pressure.

The last factor we should consider is the effect of the intermediate principal stress $\sigma_2$ on conditions at failure.
Since by definition $\sigma_2$ lies somewhere between the major and minor principal stresses, the Mohr circles for the three principal stresses look like those shown in Fig. 3c and again in Fig. 11.

It is obvious that $\sigma_2$ can have no influence on the conditions at failure for the Mohr failure criterion, no matter what magnitude it has.

The intermediate principal stress $\sigma_2$ probably does have an influence in real soil, but the Mohr-Coulomb failure theory does not consider it.
DIRECT SHEAR TEST

This test is performed to determine the consolidated-drained shear strength of a sandy to silty soil.

ASTM D 3080 - Standard Test Method for Direct Shear Test of Soils Under Consolidated Drained Conditions
This test is probably the oldest strength test because Coulomb used a type of shear box test more than 200 years ago to determine the necessary parameters for his strength equation.

The test in principle is quite simple. Basically, there is a specimen container, or “shear box,” which is separated horizontally into halves.
DIRECT SHEAR TEST

One-half is fixed; with respect to that half the other half is either pushed or pulled horizontally.

A normal load is applied to the soil specimen in the shear box through a rigid loading cap.

The shear load $T$, horizontal deformation $\delta$, and vertical deformation $\Delta H$ are measured during the test.

Dividing the shear force $T$ and the normal force $P$ by the nominal area $A$ of specimen, we obtain the shear stress $\tau$ as well as the normal stress $\sigma$ on the failure plane.
Failure plane is forced to be horizontal with this apparatus.

Fig. 12b shows some typical test results. The Mohr-Coulomb diagram for conditions at failure appears in Fig.12c.

If we were to test 3 samples of a sand at the same relative density just before shearing, then as the normal stress, was increased, we would expect from our knowledge of sliding friction a concurrent increase in the shear stress on the failure plane at failure (the shear strength).
This condition is shown in the typical shear stress versus deformation curves for a dense sand in Fig. 12b for $\sigma_{n1} < \sigma_{n2} < \sigma_{n3}$. When these results are plotted on a Mohr diagram, Fig. 12c, the angle of internal friction $f$, can be obtained.

Typical results of vertical deformation $\Delta H$ for a dense sand are shown in the lower portion of Fig. 12b.

At first there is a slight reduction in height or volume of the soil specimen, followed by a dilation or increase in height or volume.
As the normal stress $s_n$ increases the harder it is for the soil to dilate during shear, which seems reasonable.

We do not obtain the principal stresses directly in the direct shear test.

Instead, if they are needed, they may be inferred if the Mohr-Coulomb failure envelope is known.

Then the angle of rotation of the principal stresses may be determined.
Why is there rotation of the principal planes?

Initially, the horizontal plane (potential failure plane) is a principal plane (no shear stress), but after the shearing stress is applied and at failure, by definition, it cannot be a principal plane.

Therefore, rotation of the principal planes must occur in the direct shear test.

How much do the planes rotate?

It depends on the slope of the Mohr failure envelope, but it is fairly easy to determine, if you make some simple assumptions.
There are, of course, several advantages and disadvantages of the direct shear test.

**ADVANTAGES**: the test is inexpensive, fast, and simple, especially for granular materials.

We do observe shear planes and thin failure zones in nature, so it seems alright to actually shear a specimen of soil along some plane to see what the stresses are on that plane.

**DISADVANTAGES** include the problem of controlling drainage, it is very difficult if not impossible, especially for fine-grained soils.

Consequently, the test is not so suitable for other than completely drained conditions.
**DIRECT SHEAR TEST**

When we force the failure plane to occur, how can we be sure that it is the weakest direction or even at the same critical direction as occurs in the field?

We don’t know. Another flaw in the direct shear test is that there are rather serious stress concentrations at the sample boundaries, which lead to highly nonuniform stress conditions within the test specimen itself.

And finally an uncontrolled rotation of principal planes and stresses occurs between the start of the test and failure.

To accurately model the in situ loading conditions, the amount of this rotation should be known and accounted for, but it isn’t.
(1) Weigh the initial mass of soil in the pan.
(2) Measure the diameter and height of the shear box.
(3) Carefully assemble the shear box and place it in the direct shear device. Then place a porous stone and a filter paper in the shear box.
(4) Place the sand into the shear box and level off the top. Place a filter paper, a porous stone, and a top plate (with ball) on top of the sand.

(5) Remove the large alignment screws from the shear box. Open the gap between the shear box halves to approximately 0.5 mm using the gap screws, and then back out the gap screws.

(6) Weigh the pan of soil again and compute the mass of soil used.
(7) Complete the assembly of the direct shear device and initialize the three gauges (horizontal displacement gage, vertical displacement gage and shear load gage) to zero.

(8) Set the vertical load (or pressure) to a predetermined value, and then close bleeder valve and apply the load to the soil specimen by raising the toggle switch.
(9) Start the motor with selected speed so that the rate of shearing is at a selected constant rate, and take the horizontal displacement gauge, vertical displacement gage and shear load gage readings. Record the readings on the data sheet. (Note: Record the vertical displacement gage readings, if needed).

(10) Continue taking readings until the horizontal shear load peaks and then falls, or the horizontal displacement reaches 15% of the diameter.
DIRECT SHEAR TEST – analysis

(1) Calculate the density of the soil sample (mass of soil/volume of the box).

(2) Compute the cross sectional sample area \( A \) and the vertical stress \( \sigma \)

(3) Calculate shear stress \( \tau \) using:

\[
\tau = \frac{T}{A}
\]

\[
\sigma = \frac{N}{A}
\]

Where \( T \) = shear load measured with shear load gage

(4) Plot the graph with shear stress \( (\tau) \) vs. horizontal displacement \( \delta \)

(5) Calculate the maximum shear stress \( \tau_{\text{max}} \) for each test.

(6) Plot the value of the maximum shear stress \( \tau_{\text{max}} \) versus the corresponding vertical stress \( \sigma \) for each test, and determine the angle of shear strength \( \phi \) from the slope of the approximated Mohr-Coulomb failure envelope.
During the early history of soil mechanics, the direct shear test was the most popular shear test. Then, about 1930, A. Casagrande began research on the development of cylindrical compression tests in an attempt to overcome some of the serious disadvantages of DS test.

Now this test, called **triaxial test**, is by far the more popular of the two.

Triaxial test is much more complicated than the direct shear but also much more versatile.

We can control drainage quite well, and there is no rotation of $\sigma_1$ and $\sigma_3$.

Stress concentrations still exist, but they are significantly less than in the direct shear test.

Also, the failure plane can occur anywhere.
Triaxial apparatus
An added advantage: we can control the stress paths to failure reasonably well, which means that complex stress paths in the field can more effectively be modelled in the laboratory with the triaxial test.

The principle of the triaxial test is shown in Fig.13a.

The soil specimen is usually encased in a rubber membrane to prevent the pressurized cell fluid (usually water) from penetrating the pores of the soil.
TRIAXIAL TEST

Axial load is applied through a piston, and often the volume change of the specimen during a drained test or the induced pore water pressure during an undrained test is measured.

As mentioned above, we can control the drainage to and from the specimen, and it is possible, with some assumptions, to control the stress paths applied to the specimen.

Basically, we assume the stresses on the boundary of the specimen are principal stresses (Fig.13b).

Figure 13
This is not really true because of some small shear stresses acting on the ends of the specimen.

Also the failure plane is not forced, the specimen is free to fail on any weak plane or, as sometimes occurs, to simply bulge.

You will note that the $\sigma_{\text{axial}}$ in Fig.13b is the difference between the major and minor principal stresses; it is called the principal stress difference (or sometimes deviator stress).
Note also that for the conditions shown in the figure, \( \sigma_2 = \sigma_3 = \sigma_{\text{cell}} \).

Sometimes we will assume that \( \sigma_{\text{cell}} = \sigma_1 = \sigma_2 \) for special types of stress path tests.

Common triaxial stress paths are discussed in the next section.

The triaxial test is far more complex than the direct shear test.

Drainage conditions or paths followed in the triaxial test are models of specific critical design situations required for the analysis of stability in engineering practice.
These are commonly designated by a two letter symbol.

The first letter refers to what happens before shear that is, whether the specimen is consolidated.

The second letter refers to the drainage conditions during shear.

The three permissible drainage paths in the triaxial test are as follows:

<table>
<thead>
<tr>
<th>drainage path before shear/during shear</th>
<th>symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>unconsolidated - undrained</td>
<td>UU</td>
</tr>
<tr>
<td>consolidated - undrained</td>
<td>CU</td>
</tr>
<tr>
<td>consolidated - drained</td>
<td>CD</td>
</tr>
</tbody>
</table>
A conventional CD triaxial test is conducted on a sand. The cell pressure is 100 kPa; the applied axial stress at failure is 200 kPa.

**Required:**

- Plot the Mohr circles for both the initial and failure stress conditions;
- Determine $\phi$, (assume $c = 0$).
- Determine the shear stress on the failure plane at failure $t_{ff}$ and find the theoretical angle of the failure plane in the specimen. Also determine the orientation of the plane of maximum obliquity.
- Determine the maximum shear stress at failure $\tau_{max}$ and the angle of the plane on which it acts; calculate the available shear strength on this plane and the factor of safety on this plane.