Soil Shear Strength – The Friction Model

Lecture No. 9
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Shear Deformation of Soils

- Let us consider two different types of soil – **Type 1** and **Type 2** – and subject undisturbed samples of these two soils to **shear deformation** as shown in the figure below:

- **Type 1** soils include loose sands and soft clays and **Type 2** soils include dense sands and stiff clays.

Shear Deformation (Continued..)

- This type of shear deformation is called **simple shear** deformation and the testing apparatus is called the **simple shear apparatus**.

- Of course, we could have used the **direct shear apparatus** shown below or a **triaxial apparatus** to apply shear deformations.

- The observations that we are about to discuss would have been the same regardless of the testing apparatus used.

Response of Soils to Shearing

- **Type 1** soils:
  - show gradual increase in shear stress as the shear strain increases until an approximately constant shear stress – **critical state shear stress** – is attained.
  - undergo contraction or compressive volumetric strains, i.e. they become **denser** as the shear strain increases, until a point when the shearing continues at **constant volume**.
Response of Soils (Continued..)

- **Type 2** soils:
  - show rapid **increase** in shear stress, reaching a **peak** value at **low shear strains** and then show a **decrease** in shear stress with increasing shear strain, ultimately attaining a **critical state shear stress**.
  - compress initially due to particle adjustment and then expand or dilate, i.e. they become looser as the shear strain increases, until a point when the shearing continues at **constant volume**.

Critical Void Ratio

- The void ratio of **Type 1** soils is **fairly high** and that of **Type 2** soils is **fairly low** before shearing.
- During shearing, **Type 1** soils contract while **Type 2** soils dilate.
- Therefore, the void ratio of **Type 1** soil **decreases** and that of **Type 2** soil **increases** during shearing.
- At **large shear strains**, the volume of the soil mass remains **constant**.

Effect of Normal Effective Stress

- If we conducted several tests on each of the two types of soils at **different normal effective stress** ($\sigma_z$), we observe stress-strain responses as shown in the figure on the left.
- **Higher** normal effective stresses result in **higher critical state** and **peak** shear stresses and **lower** critical void ratios.
- **Dilation** of **Type 2** soils is **suppressed** at high effective normal stresses.

Normal Effective Stress (Continued..)

- We can tabulate the readings of **peak** and **critical state** shear stresses along with the effective normal stress in each test and plot them in a shear stress ($\tau$) – normal effective stress ($\sigma'_n$) space as shown in the figure.
- For **Type 1** soils, there is no peak shear stress; the critical state shear stresses plot on a **straight line passing through the origin**.
- For **Type 2** soils, the critical state shear stresses coincide with those of **Type 1** soils but the peak shear stresses plot on a **curve** as shown above.
The Friction Model

- Consider the slip of a wooden block in response to a horizontal force $H$ as shown in the figure.
  \[ H = \mu W \text{ or } T = \mu N \]
  where $\mu$ is the coefficient of friction between the table and the block. This is the Coulomb’s law of friction.
- In terms of stresses, we can use Coulomb’s law as follows:
  \[ \tau_f = (\sigma_n') \tan \phi' \]
  where $\tau_f$ is the shear stress at slip and $(\sigma_n)'$ is the normal effective stress on the plane.

The Friction Model (Continued..)

- Coulomb’s law requires the existence or the development of a slip plane or a failure plane.
- In the case of the wooden block on the table, the slip plane is the horizontal interface between the block and the table.
- For a conglomerate of soil particles, we do not know where the slip plane is going to be.
- However, we can guess the direction of the slip plane from the shear test results plotted in the shear stress – normal stress space.

Extended Friction Model

- Let’s consider an idealized shear model of a Type 1 soil as shown in the figure on the right.
- For a loose assembly of spherical particles of Type 1 soil, sliding would be initiated on the horizontal plane a-a, consistent with our Friction Model.
- Once in motion, the particles would tend to move into the void spaces.
- The direction of motion would have a downward component, indicating contraction.
Extended Friction Model (Continued..)

• An idealized shear model of a Type 2 soil is shown in the figure on the right.

• In a dense assembly of spherical particles of Type 2 soil, relative horizontal sliding of Row 2 with respect to Row 1 is restrained by interlocking of the particles.

• Sliding can only be initiated on an inclined plane.

• Particles must ride up over each other or be pushed aside or both.

• The direction of motion would have an upward component indicating dilation.

Extended Friction Model (Continued..)

• An Extended Friction Model that can incorporate the peak behaviour of Type 2 soils can be formulated if we modified our wooden block analogy.

• Instead of being pushed on a horizontal plane, we will consider the wooden block being pushed up a plane inclined at an angle $\alpha$ with respect to the horizontal.

Extended Friction Model (Continued..)

• Considering equilibrium in X- and Z-directions and solving for $W$ and $H$, we get:

$$\begin{align*}
H &= \tan \phi' + \tan \alpha \\
W &= 1 - \tan \phi' \tan \alpha
\end{align*}$$

• Dividing $H$ and $W$ by the contact area $A$, we obtain the equation for the Extended Friction Model as:

$$\tau_f = \left(\sigma'_n\right)_f \tan \phi' + \tan \alpha = \left(\sigma'_n\right)_f \tan(\phi' + \alpha)$$

Extended Friction Model (Continued..)

• The equation for the Extended Friction Model is:

$$\tau_f = \left(\sigma'_n\right)_f \tan(\phi' + \alpha)$$

• The magnitude of $\alpha$ depends on the magnitude of the normal effective stress.

• We have seen in the shear behaviour of Type 2 soils (page 7) that the dilation is suppressed at high normal effective stresses.

• Therefore, at low normal effective stresses, $\alpha$ takes high values and this results in the curved envelope of peak shear stresses as shown above.