Stresses and Strains

Lecture No. 6

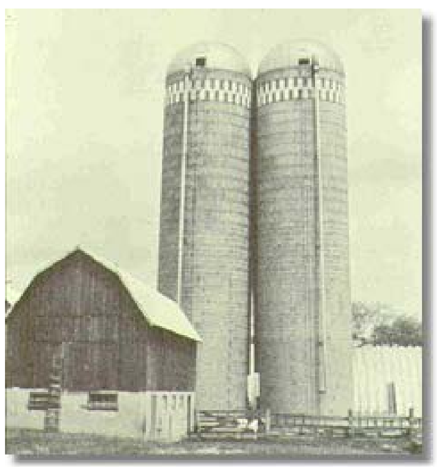
October 01, 2002

Determination of Stresses and Strains – Why?

- A geotechnical engineer must ensure that a geotechnical structure
  - must not collapse under any anticipated loading conditions
  - must not settle beyond a certain tolerable limit
- Calculation of collapse load and maximum settlement require that the stresses and strains within the soil layer be determined.
- Spectacular failures that are often catastrophic, have occurred due to a lack of proper understanding of the stresses and strains within the soil layer.

The Case of the "Kissing" Silos

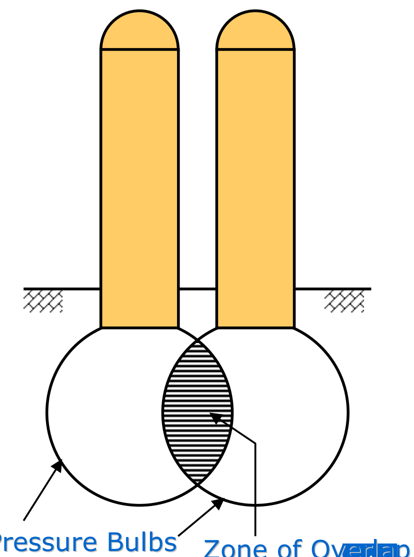
- Complete disregard for accurate estimation of stresses and the resulting strains in the foundation soil layer for these two silos resulted in their construction so close to each other.
- The result was spectacular as shown in the picture.



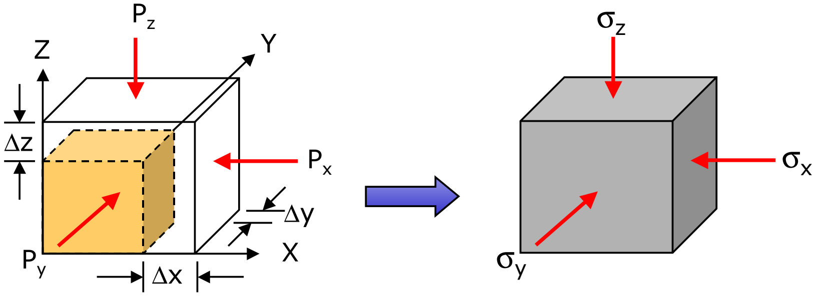
The "Kissing" Silos

"Kissing" Silos (Continued..)

- By deciding to put the two silos close to each other, the engineer ensured that the zone of influence of stresses imposed by the two silos – **Pressure Bulbs** – overlapped significantly as shown in the figure.
- As a result, the soil in the zone of overlap experienced **much higher stresses** and duly obliged by **settling much more**.



Normal Stresses and Strains



- Consider a cube of dimensions **x**, **y** and **z** that is subjected to normal forces $\mathbf{P_x}$, $\mathbf{P_y}$ and $\mathbf{P_z}$ as shown above.
- The **normal stresses** are:

$$\sigma_x = \frac{P_x}{yz}, \sigma_y = \frac{P_y}{xz}, \sigma_z = \frac{P_z}{xy}$$

- Let us assume that under these three forces the cube compresses by Δ**x**, Δ**y** and Δ**z** in X-,Y- and Z-directions.
- The **normal strains** are:

$$\varepsilon_x = \frac{\Delta x}{x}, \varepsilon_y = \frac{\Delta y}{y}, \varepsilon_z = \frac{\Delta z}{z}$$

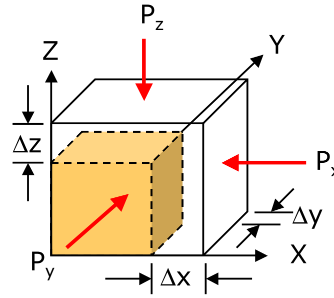
Volumetric Strain

- Considering the same cube, the **original volume** of the cube was:

$$V_o = xyz$$

- The **new volume** of the cube after compression is:

$$V_1 = (x - \Delta x)(y - \Delta y)(z - \Delta z)$$



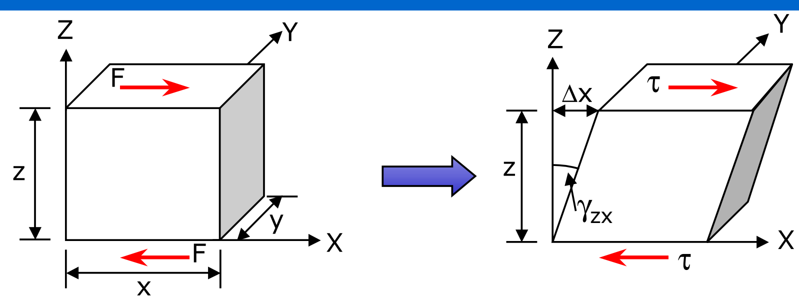
- The **change in volume** is $\Delta \mathbf{V} = \mathbf{V_o} - \mathbf{V_1}$ and the **volumetric strain** is defined as:

$$\varepsilon_v = \frac{\Delta V}{V_o} = \frac{xyz - (x - \Delta x)(y - \Delta y)(z - \Delta z)}{xyz}$$

- Neglecting second and third order terms of Δ, we get:

$$\varepsilon_v = \frac{\Delta x}{x} + \frac{\Delta y}{y} + \frac{\Delta z}{z} = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

Shear Stresses and Strains



- Let's apply a couple **F** in the X-direction distorting the square in X-Z plane to into a parallelogram as shown in the figure above.
- The couple **F** is called the **shearing force** and the **shear stress** is defined as:

$$\tau = F/(xy)$$

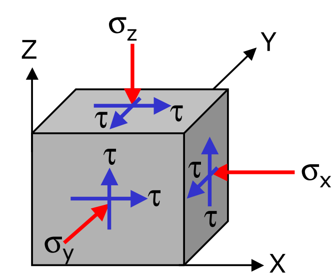
- Shear strain is a measure of the angular distortion of a body by shearing forces. It is defined as:

$$\gamma_{zx} = \tan^{-1}\frac{\Delta x}{z}$$

- For small strains, $\mathbf{tan(\gamma_{zx}) = \gamma_{zx}}$. Therefore, $\gamma_{zx} = \Delta x / z$

Principal Stresses and Strains

- Generally, any plane will be acted upon by both normal and shear stresses as shown in the figure on the right.
- If the shear stress on a plane is zero, the plane is called a principal plane and the normal stress is called the principal stress.

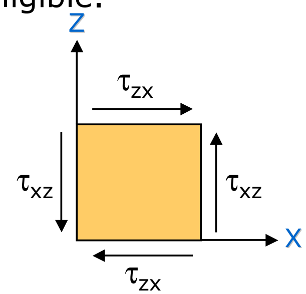


- We will discuss the concept of principal planes and principal stresses later in this topic.

Stresses and Strains – Sign Convention

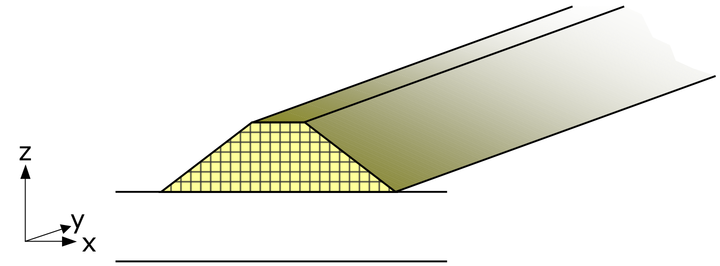
- In geotechnical engineering, **compressive** normal stresses and **compressive** strains are positive.
- Soils cannot sustain any appreciable magnitude of tensile stresses and therefore, the tensile strength of soils is generally considered negligible.
- Shear stresses are always **complementary** as shown in the figure on the right in order to satisfy the **equilibrium**, that is,

$$\tau_{zx} = \tau_{xz}$$



- Shear stresses that provide **anti-clockwise** couple are considered **positive**.

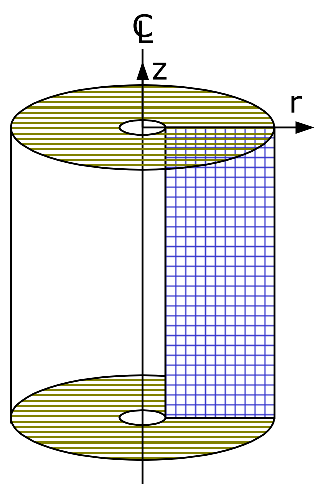
Plane Strain Condition



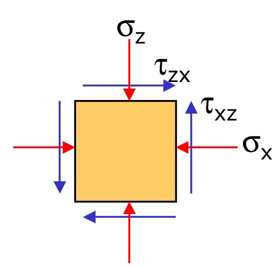
- For a **plane strain** condition, one dimension is considerably greater than the other two dimensions (in the figure shown above, the y-direction).
- Strains along the y-direction can be assumed to be **zero**.
- We only have to solve for strains in **one plane** (in this case, the **x-z** plane).
- For seepage problems, the term **plane flow** is used for a 2-D seepage analysis.

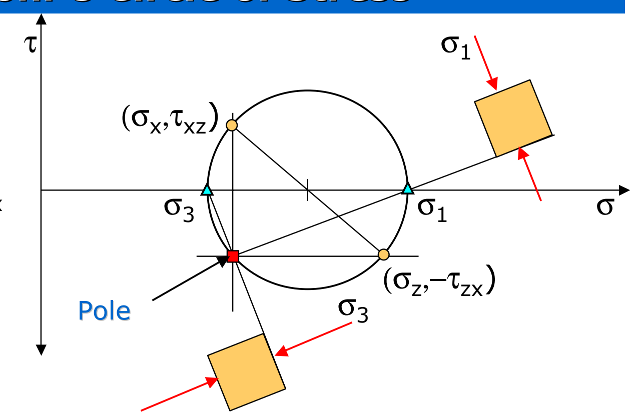
Axisymmetric Condition

- For an **axisymmetric** problem, **both** the structure and the loading exhibit **radial** symmetry about the central vertical axis.
- **Circumferential strains** can be **ignored** in the solution.
- If the loading is not symmetric about the central vertical axis, the problem is not truly axisymmetric.
- An example of a geotechnical axisymmetric problem:
  - pile foundation subject to concentric vertical load



Mohr's Circle of Stress

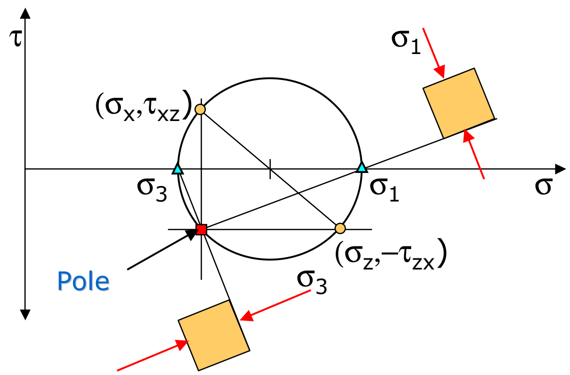




- Mohr's Circle of Stress represents a convenient way of conducting stress analysis and has been covered in the Strength of Materials course.
- The only difference here is that compressive normal stresses are taken as positive instead of tensile stresses.

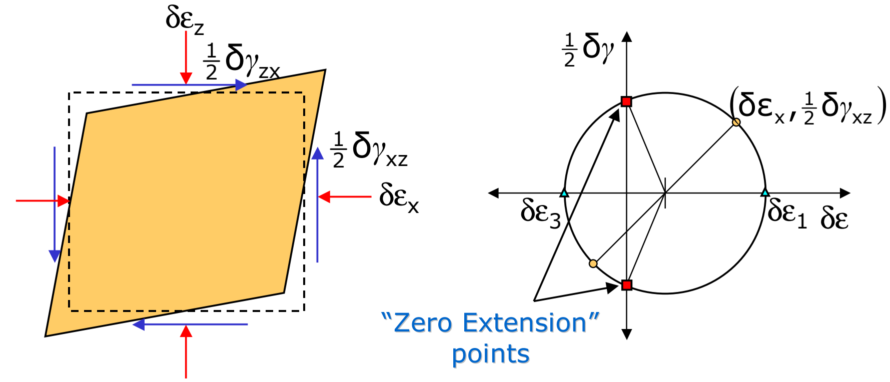
Mohr's Circle of Stress (Continued..)

- Stresses σ_1 and σ_3 lie on the normal stress axis, i.e. the shear stresses at these points are zero. Therefore, these stresses are termed as **principal stresses**.
- Since $\sigma_1 > \sigma_3$, σ_1 is called the **major** principal stress and σ_3 is called the **minor** principal stress.



- The plane on which the major principal stress σ_1 acts is called the **major principal plane**.
- Minor principal stress σ_3 acts on **minor principal plane** that is **perpendicular** to the **major principal plane**.

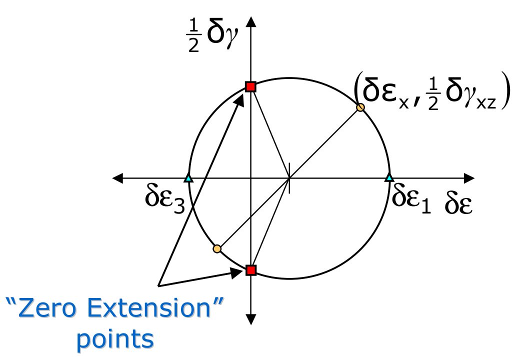
Mohr's Circle of Strain



- The procedure for construction of Mohr's Circle of Strain is similar to that for the Circle of Stress.
- Since strain has no absolute zero, increments of strain are used for plotting.

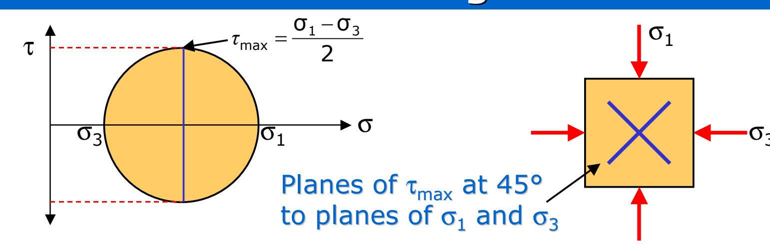
Mohr's Circle of Strain (Continued..)

- Note that the y-axis of the coordinate system for Mohr's circle of strain is represented by 0.5 times the incremental shear strain ($\delta\gamma$).
- The strains $\delta\varepsilon_1$ and $\delta\varepsilon_3$ are known as the **major** and the **minor** principal strains, respectively.



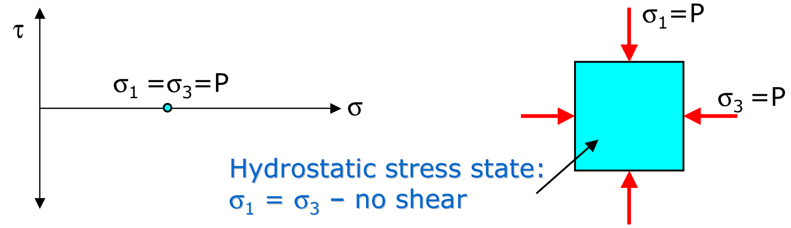
- Unlike the stresses, incremental normal strains can be **tensile** (for example, minor principal strain is $\delta\varepsilon_3$ negative and hence, tensile).
- At **zero extension points**, the normal strains are **zero**. At these points the material experiences **pure shear strains**.

Soil Strength



- In geotechnical engineering, strength may be defined as **the ability to resist shear**.
- It is the ability of a material to resist shear that enables the major and the minor principal stresses to be **different**.
- This is indicated by the Mohr's Circle of Stress for the plane containing the two principal stresses as shown above.

Soil Strength (Continued..)



- Fluids such as water **cannot sustain shear stresses** when they are stationary.
- The stress within a stationary fluid must therefore be **equal in all directions**, and the Mohr's Circle of Stress is effectively a single point (zero radius) as shown in the figure above.

Soil Strength (Continued..)

- Unless a material can withstand shear stresses, we won't be able to use it to make non-horizontal surfaces (e.g. embankments and slopes).
- Soil is able to withstand shear stresses while the water is not.
- Therefore, it is necessary to distinguish the component of stress carried by the **soil particles** from the component carried by the **pore water**.
- This is done using the **Effective Stress Principle**.

The Effective Stress Principle

- The **Effective Stress Principle** is the most important concept in geotechnical engineering.
- It states that the **total stress** carried by a saturated soil layer is the sum total of **effective stress** carried by the soil particles and the **pressure** carried by the **pore water**:

  $\sigma = \sigma' + u$ or $\sigma' = \sigma - u$

  where σ is the total stress, σ' is the effective stress and u is the pore water pressure.
- **Deformation** of a soil is a function of the **change in effective stress** and not total stress.

Effective Stress Principle (Continued..)

- The **Effective Stress Principle** applies only to **normal stresses** and not to shear stresses.
- Since pore water cannot resist shear stresses, these must be entirely resisted by the soil particles. Thus:

  $\tau = \tau'$

- Effective stress is **not** the **contact stress** between two soil particles but is the **average stress** on a plane through the soil mass as shown in the figure on the right.