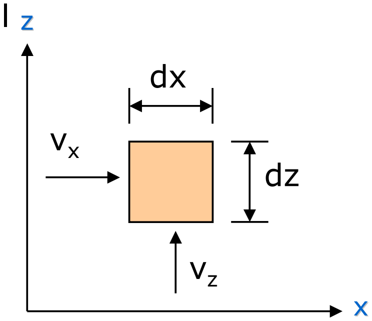
Seepage – Flow Nets

Lecture No. 4

September 19, 2002

2-D Seepage – Laplace Equation

- Considering a two-dimensional element of soil of dimensions **dx** and **dz** in the **x** and **z** directions, respectively.
- It is assumed that the soil is homogeneous and **isotropic** with respect to permeability.
- The pore fluid is assumed to be **incompressible**.



- The governing differential equation for the groundwater flow is obtained by equating the flow rates into and out of the elements.

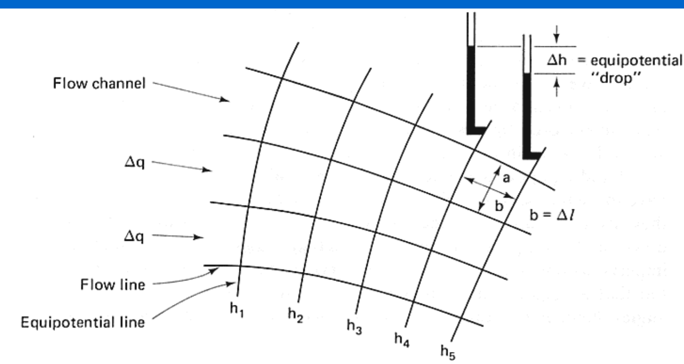
$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0$$ [Laplace's Equation]

[See Holtz and Kovacs, Appendix B for the derivation of this equation. PDF file available on the website.]

Laplace's Equation

- Laplace's equation is a very important equation in engineering physics.
- It represents the energy loss through any resistive medium:
  - Fluid flow through porous medium (e.g. soil)
  - Electron flow through a conducting material
  - Heat flow through a conducting material
  - Flow of people in and out of hospitals !
- Exact solution of Laplace's equation for 2-D seepage can be obtained for cases with simple boundary conditions.
- For most practical geotechnical problems, it is simpler to solve this equation graphically by drawing **flow nets**.

Flow Nets



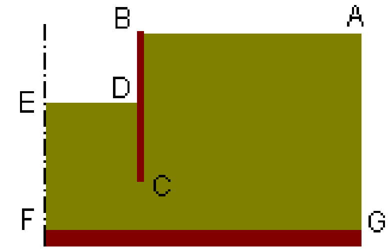
- A flow net consists of two sets of curves – **equipotentials** and **flow lines** – that intersect each other at 90°.
- Along an equipotential, the total head is **constant**.
- A pair of adjacent flow lines define a **flow channel** through which the **rate of flow** of pore fluid is **constant**.
- The loss of head between two successive equipotentials is called the **equipotential drop**.

Flow Net – Sketching Rules

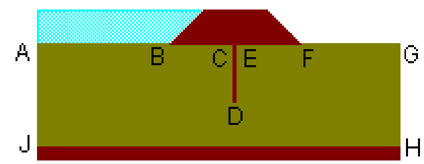
- Flow lines cross the equipotentials at right angles. By definition, there is **no flow** along an equipotential and therefore, all of the flow must be at 90° to it.
- A flow line **cannot cross other flow lines**: two molecules of water cannot occupy the same space at the same time.
- An equipotential **cannot cross other equipotentials**: one point cannot have two different values of total head.

Flow Nets – Sketching Rules (continued..)

- Impermeable boundaries and lines of symmetry are flow lines: as there is no flow across them, all of the flow must be along them. For example, lines **EF** and **FG** in the figure on the right are **flow lines**.

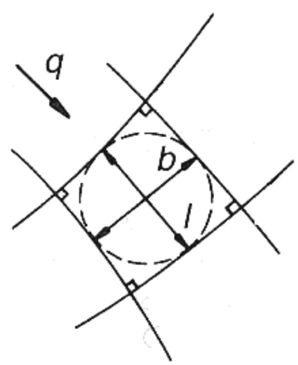


- Bodies of water, such as reservoirs behind a dam, are equipotentials. For example, line **AB** in the figure on the right is an **equipotential**.

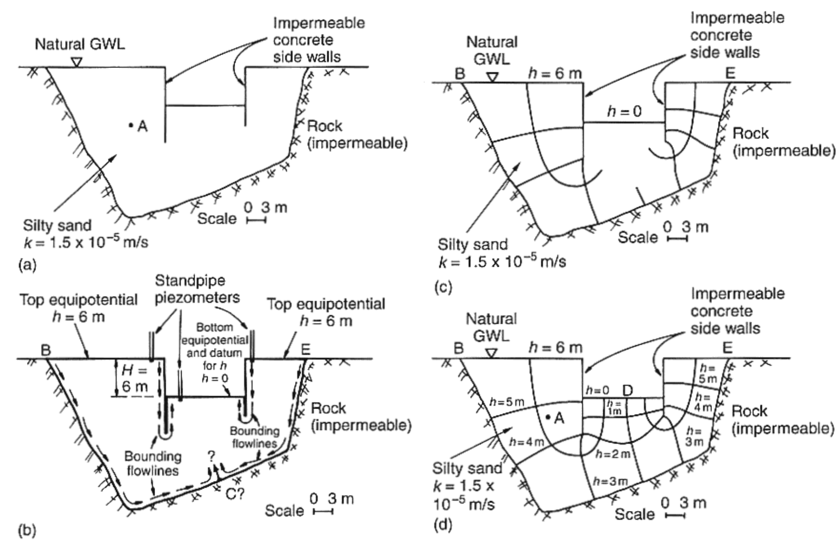


Flow Nets – Sketching Rules (continued..)

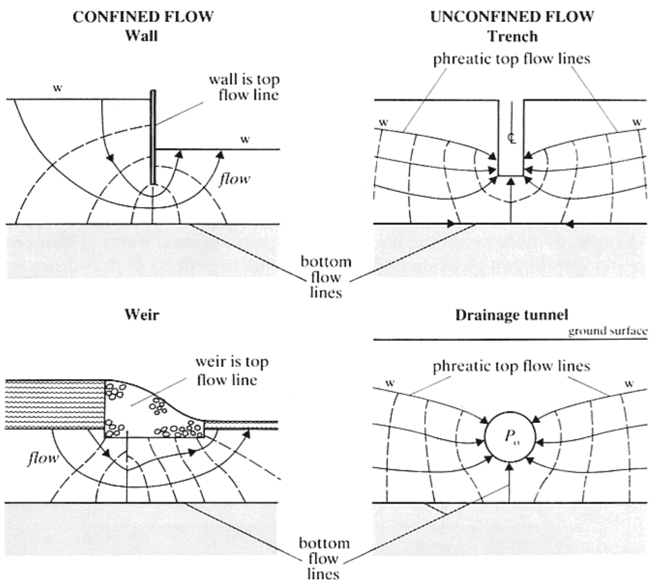
- Although an infinite number of flow lines and equipotentials could be sketched, the flow net must be constructed so that each element is a **curvilinear square**. Although its sides may be curved, a curvilinear square is **as broad as it is long**, so that **a circle** may be inscribed within it that **touches all four of its sides** as shown in figure on the right.



Flow Net Sketching – An Example

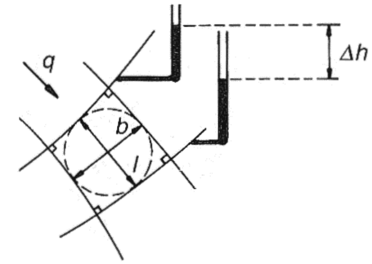


Typical Flow Nets



Flow Rate Calculation using a Flow Net

- Consider groundwater flow through a single flow element shown in the figure on the right.
- The flow rate through this element, **q**, is given by:



$$q = k \cdot i \cdot A = k \cdot \frac{\Delta h}{l} \cdot b$$

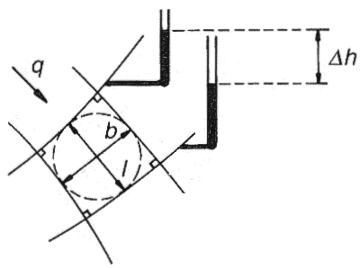
[k – permeability of soil]

- If the element is a curvilinear square, i.e. **b = l**, the above equation reduces to:

$$q = k \cdot i \cdot A = k \cdot \Delta h$$

Flow Rate Calculation (continued..)

- For $\mathbf{N_F}$ number of flow channels, $\mathbf{N_H}$ number of equipotential drops and an overall head drop of **H**:



$$q_T = q \cdot N_F \quad \text{and} \quad \Delta h = \frac{H}{N_H}$$

- Therefore, the expression for **flow rate per unit length**, q_T, can be obtained as:

$$q_T = k \cdot H \cdot \left(\frac{N_F}{N_H}\right)$$

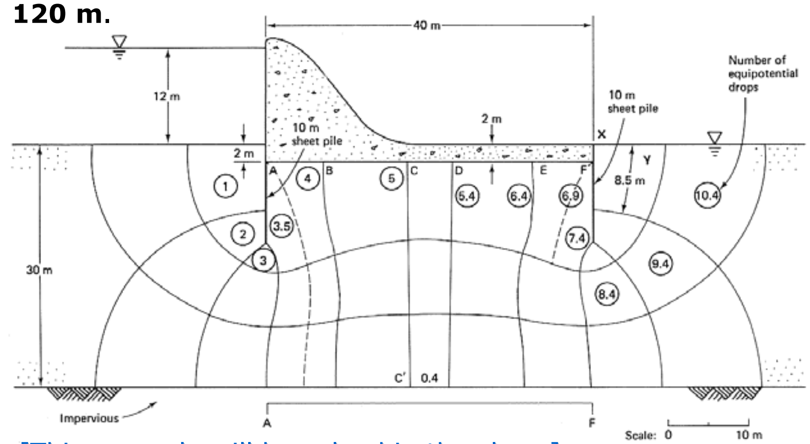
- The **total flow rate**, Q_T, is given by:

$$Q_T = q_T \cdot L$$

[L is the length perpendicular to the 2-D seepage plane.]

Flow Rate Calculation – An Example

- For the concrete dam shown below, calculate the quantity of seepage flow rate under the dam when the permeability of the soil, **k**, is equal to **2x10^{-3}** cm/s. Length of the dam is **120 m**.



[This example will be solved in the class.]

Pore Pressure Calculation using Flow Net

- A flow net can be used for the calculation of pore pressure at any point in the soil that lies within the domain of the flow net.
- First, the number of equipotential drops from a known total head to the given point are estimated from the flow net. This gives the total head at the given point as:

$$h_t = H - H \cdot (n_H / N_H)$$

[H – known total head; N_H – total no. of equipotential drops; n_H – no. of equipotential drops to the given point.]

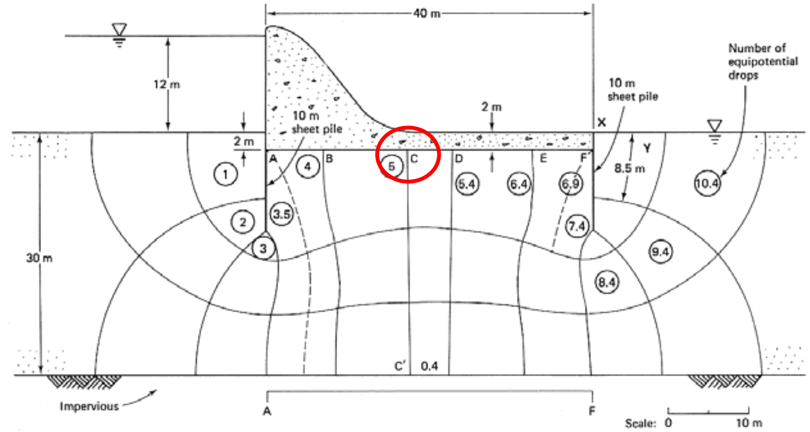
- Then the elevation head at the given point with respect to the datum is estimated from the geometry of the problem.
- The pressure head and pore pressure at the given point can then be calculated as:

$$h_p = h_t - h_e$$ and $$u_p = h_p \cdot \rho_w \cdot g = h_p \cdot \gamma_w$$

[ρ_w is the density and γ_w is the unit weight of water; g is Earth's gravitational acceleration.]

Pore Pressure Calculation – An Example

- For the concrete dam shown below, calculate the pressure at point C.



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