Permeability

Lecture No. 3

September 17, 2002

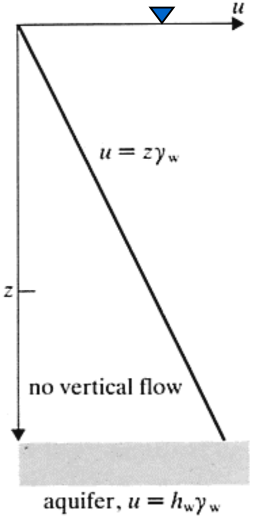
Soil Moisture and Groundwater

- Water accumulating on the surface of pervious soils or rocks percolates into the ground due to gravity.
- It joins existing groundwater and flows laterally towards major surface or subsurface drainage sinks such as rivers, lakes, rock faults, buried pervious channels.
- This causes seasonal changes in the groundwater level depending on the climatic conditions.
- The quantity of groundwater flow depends on the **permeability** of the earth materials.
- The permeability of the earth material depends primarily on the **average size** of the pores and their **inter-connectivity**.

Hydrostatic Groundwater Condition

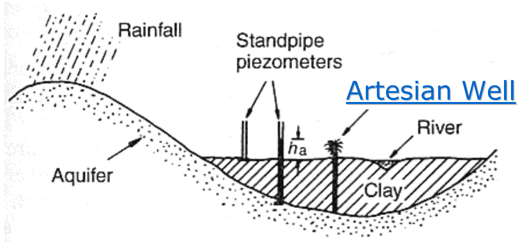
- A **hydrostatic** groundwater condition shown in the figure on the left is characterized by **no water flow** in any direction.
- The pore water pressures increase **linearly** with depth below the groundwater surface or water table.
- The pore water pressure at any depth z below the water table is given by:

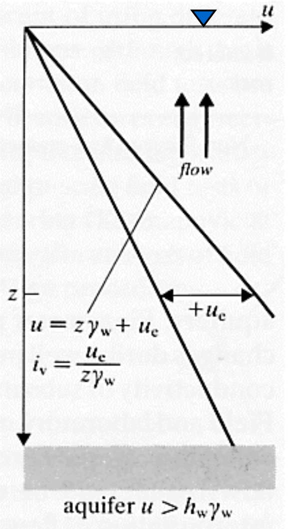
$$\boxed{u_h = z \cdot \gamma_w}$$



Artesian Groundwater Condition

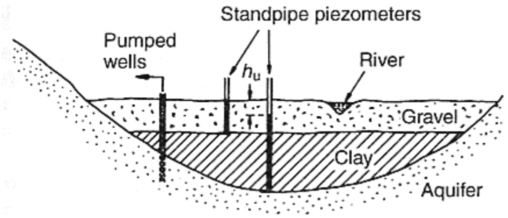
- Figure below shows an example of an **artesian** groundwater condition.
- Pore pressure at any depth z is **higher** than hydrostatic and there is **upward** water flow.

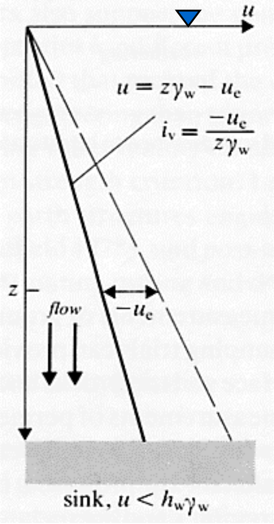




Underdrained Groundwater Condition

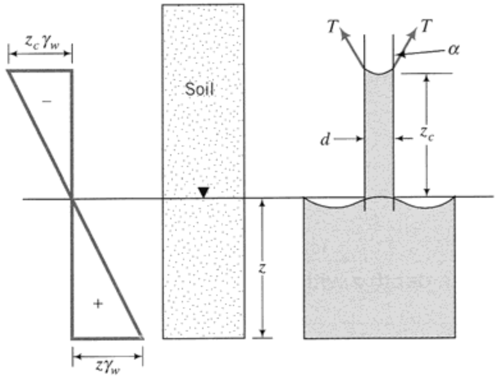
- Figure below shows an example of an **underdrained** groundwater condition.
- The pore pressure at any depth z is **smaller** than hydrostatic and there is **downward** water flow.





Porewater Pressure above Water Table

- Porewater pressure above the water is negative due to capillary action.
- Interconnected voids in the soil perform as several capillary tubes, allowing the water menisci to rise above the groundwater table.
- Capillary rise z_c is higher when diameter **d** of the tube is smaller.



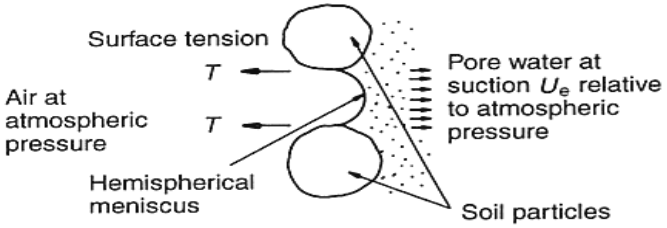
Porewater Pressure Distribution

Idealization

Air Entry Value

- There is a limit to the negative pore water pressure a soil can sustain without drawing in air through any surface exposed to the atmosphere. This limiting negative pore water pressure is known as the **air entry value**.
- Air entry value for a soil **increases** if the soil pore size **decreases**.
- The air entry value $\mathbf{U_c}$ for a soil can be estimated by considering the equilibrium of a hemispherical water meniscus in a circular pore of diameter **d**.

Air Entry Value (continued..)



Force due to surface tension around the rim of the meniscus = Force due to difference in pore water and air pressure

$$\pi dT = \left(\pi d^2/4\right)U_e$$

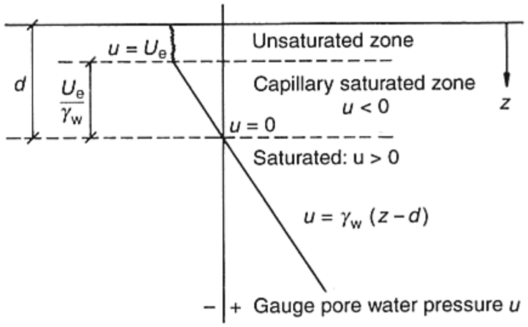
or

$$U_e = 4T/d$$

Typical Air Entry Values for Soils

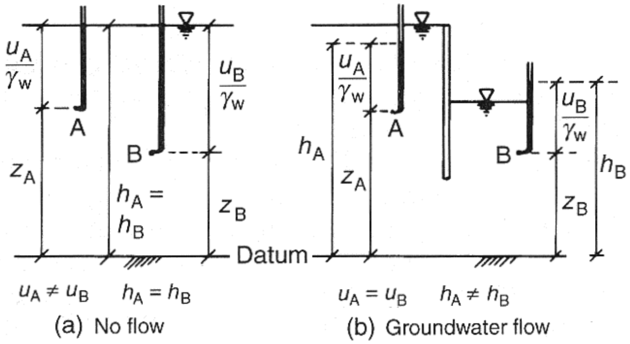
Soil Type	D_{10} (mm)	Air Entry Value (kPa)
Coarse Sand	1	0.28 to 1.4 kPa
Clay	0.001	280 to 1400 kPa

Capillary Rise above Groundwater Table



- Due to low air entry values, coarse-grained soils remain **unsaturated** above the groundwater table with very little water retained by capillary action.
- Fine-grained soils, however, may remain saturated for **several meters** above the water able with pore water pressure continuing to decrease until air entry value is reached.

Groundwater Flow

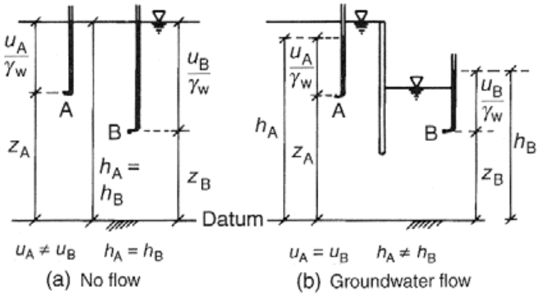


(a) No flow (b) Groundwater flow

- Consider two manometers installed in the ground at points **A** and **B** as shown in the figure above.
- For stationary groundwater, levels in both manometers will be the same as shown in figure (a).
- Water will flow from **A** to **B** if there is a difference between the water levels in the manometers as shown in figure (b).

Total, Pressure and Elevation Head

- The height of water in the manometer, measured from a certain fixed datum, is called **total head** and is equal to the sum of **pressure head** and **elevation head**. For figure (a) on the left:



(a) No flow (b) Groundwater flow

$$h_A = u_A/\gamma_w + z_A = h_B = u_B/\gamma_w + z_B$$

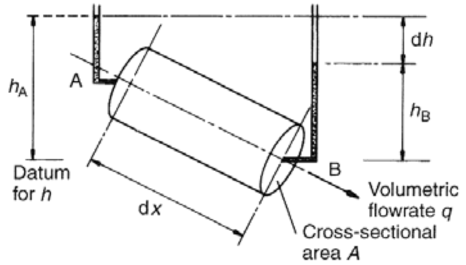
- Groundwater flow can only take place when there is a difference in **total head**.

Darcy’s Law

- Referring to the figure below, Darcy’s law can be written as:

$$q = k \cdot i \cdot A$$

where **q** is the volumetric flow rate of water, **A** is the cross-sectional area of the flow-tube, **i** is the hydraulic gradient, and **k** is the coefficient of permeability of the soil.



- Hydraulic gradient (**i**) is given by: $i = -(dh/dx)$

[Negative sign indicates that the total head (**h**) decreases in the direction of the flow.]

Coefficient of Permeability

- The **Coefficient of Permeability** (**k**) has units of velocity – **m/s**.
- It depends primarily on:
  - The size distribution and the shape of soil particles
  - Soil Structure
  - Viscosity of the pore fluid
- In general, the **smaller** the average pore size, the **lower** is the value of **k**.
- For a given soil, **k** is a function of its void ratio (**e**).
- If a soil has been deposited in layers, **k** is higher for groundwater flow parallel to the layers than that for flow across the layers.

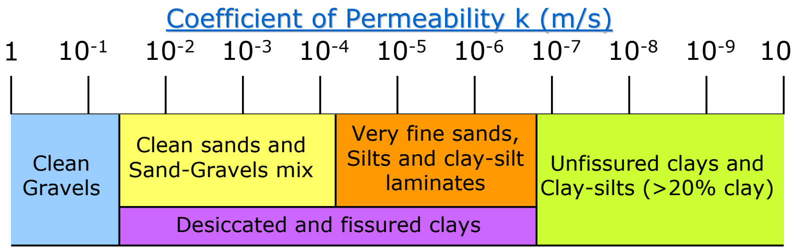
Intrinsic Permeability

- **Intrinsic Permeability** (**K**) of soil depends only on the characteristics of the soil skeleton.
- It is not affected by factors that influence the coefficient of permeability (**k**).
- The units of intrinsic permeability (**K**) are those of an area – **m²**.
- The coefficient of permeability (**k**) is related to intrinsic permeability (**K**) using the following equation:

$$\boxed{k = K(\gamma_w/\eta)}$$

where η is the dynamic viscosity of the pore fluid.

Permeability – Typical Values



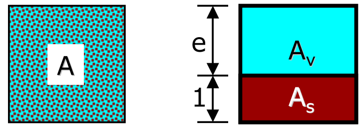
- For clean sands, **k** (in m/s) can be estimated approximately using **Hazen's formula**:

$$\boxed{k = 10^{-2} D_{10}^2}$$

where **D_{10}** is the effective particle size in **mm** (obtained from grain size distribution curve).

Velocity of Groundwater Flow

- Darcy's law is often expressed in terms of the velocity of groundwater flow as:



$$\boxed{v = k \cdot i}$$

$$\boxed{v = k \cdot i = \frac{q}{A}}$$

$$\boxed{v' = \frac{q}{A_v} = \frac{v(1+e)}{e} = \frac{v}{n}}$$

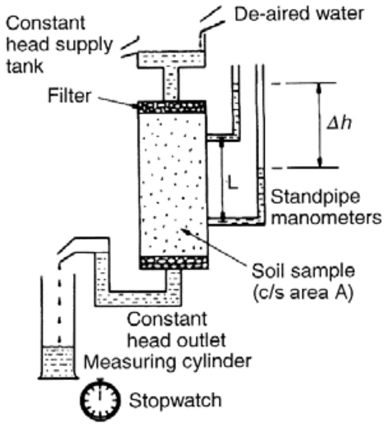
where v is known as the **superficial** or **Darcy flow velocity** and is calculated by dividing the volumetric flow rate **q** by the total area **A.**

- The **true seepage velocity** (**v'**) is obtained by dividing the flow rate **q** by the cross-sectional area of the voids alone (**A_v**) as shown in the figure above.

Laboratory Determination of k

Constant Head Permeability Test

- It is suitable only for coarse-grained soils.
- The soil is contained within a Perspex tube with inlet and outlets and filters at the top and bottom.
- The hydraulic gradient **i** for a flow rate **q** is determined from the head difference **Δh** indicated by the manometers inserted at two points at a distance **L** apart along the direction of the flow.
- The flow rate **q** is determined using a measuring cylinder and a stop-watch.
- The value of **k** is calculated as:



$$k = q/[A \cdot (\Delta h/L)]$$

Constant Head Permeability Test – An Example

- The table below gives data from a constant head permeability test on a sample of dense sand in upward flow. Plot a graph of flow rate **q** vs. hydraulic gradient **i**, and estimate the initial permeability of the sample. Cross-sectional area of the sample is **8000 mm²**.

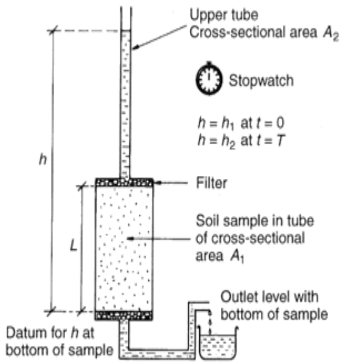
i	0.0	0.2	0.4	0.6	0.8
q (cm³/s)	0.00	1.00	2.20	3.75	5.80

[This example will be solved in the class.]

Laboratory Determination of k

Falling Head Permeability Test

- This test is more suitable for fine-grained soils.
- Water flows from a small-bore tube of cross-sectional area A_2, through the soil sample contained in a larger tube of cross-sectional area A_1.
- At time t = 0, the water level in the upper tube is at height h_1 above the permeameter outlet.
- The water level in the upper tube then falls as water flows through the soil sample. At time t = T, it falls to a height h_2 above the permeameter outlet.
- The value of k can be calculated as:



$$k = 2.303\left(\frac{A_2 L}{A_1 T}\right)\log\left(\frac{h_1}{h_2}\right)$$

[See Craig p.42 for a derivation of the above equation.]

Falling Head Permeability Test – An Example

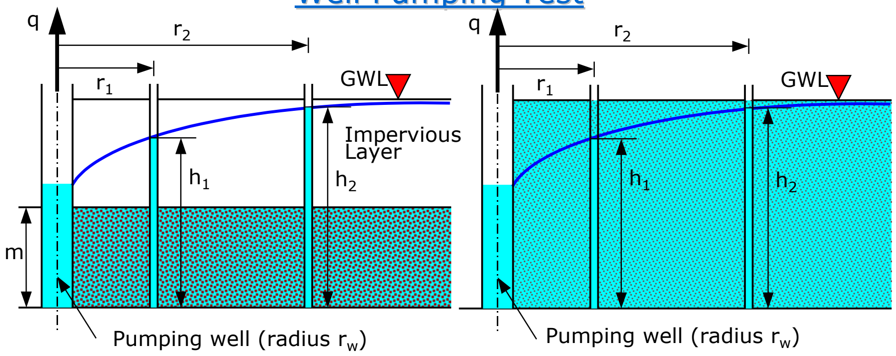
- In an attempt to investigate the overall vertical permeability of a layered deposit, an engineer carries out a falling head permeability test on an artificial sample comprising **100 mm** of silt overlying **100 mm** of sand. The results of this test are tabulated below. Estimate the overall vertical permeability at the start and at the end of the test. Given:
  - C/s area of sample $\mathbf{A_1}$ = **8000 mm²**
  - C/s area of the top tube $\mathbf{A_2}$ = **10 mm²**
  - Overall sample length = **200 mm**

t (s)	0	40	100	190	330	600
h (m)	1	0.85	0.70	0.55	0.40	0.25

[This example will be solved in the class.]

In-situ Determination of k

Well Pumping Test



Confined Layer

Unconfined Layer

$$k = \frac{2.3 q \log(r_2/r_1)}{2\pi m (h_2 - h_1)}$$

$$k = \frac{2.3 q \log(r_2/r_1)}{\pi (h_2^2 - h_1^2)}$$

[Read Craig p.42-43 for the well pumping test procedure.]

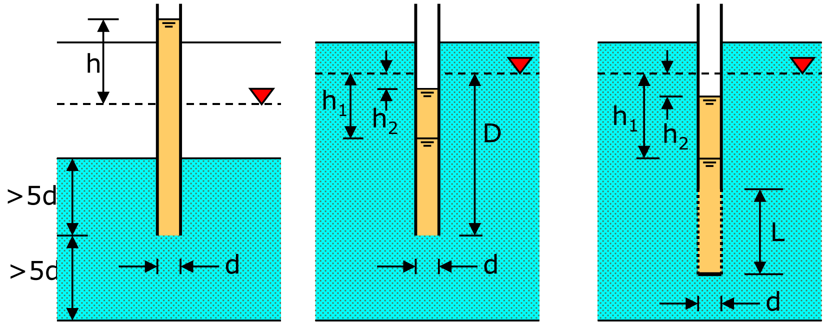
Well Pumping Test – An Example

- A well pumping test was carried out in a soil layer of thickness **15 m** to determine its coefficient of permeability. The rate of pumping was **10.6 litres/s**. Drawdowns in observation wells located at **15 m** and **30 m** from the centre of the pumping well were **1.6 m** and **1.4 m**, respectively, from the initial groundwater table located at **1.9 m** below the ground level. Calculate the coefficient of permeability of the soil layer.

[This example will be solved in the class.]

In-situ Determination of k

Borehole Tests



[Spherical Tip]

[Cylindrical Tip]

Constant Head

Falling Head

$$k = \frac{q}{2.75 (d \times h)}$$

$$k = \frac{\pi d}{11 t} \ln\left(\frac{h_1}{h_2}\right)$$

$$k = \frac{d^2}{8 L t} \ln\left(\frac{2L}{d}\right) \ln\left(\frac{h_1}{h_2}\right)$$

[Read Craig p.44-46 for the borehole test procedure.]

Borehole Test – An Example

- In order to measure the coefficient of permeability of a silty sand layer, a **200 mm** diameter borehole was drilled to a depth of **10 m**. Steel casing was installed at top **9 m** of the borehole and a perforated section was installed at the bottom **1 m**. The groundwater table was located at **2.1 m** below the ground surface. A **falling head** permeability test was then conducted in which the water level in the borehole was found to rise from a depth of **5.0 m** below ground surface to a depth of **3.9 m** below ground surface in **10 min**. Compute the permeability of the silty sand layer.

[This example will be solved in the class.]